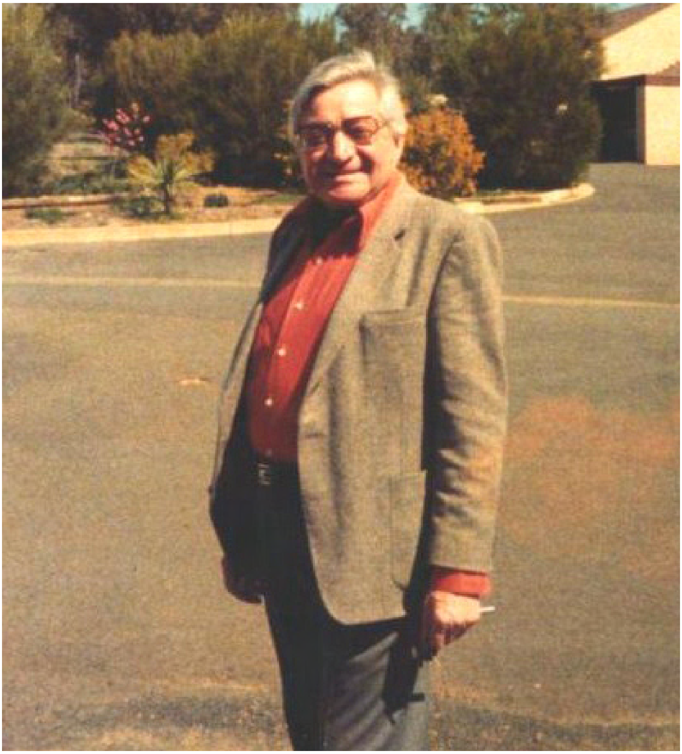
Chapter 8. The Reflective Years

Figure 8.1. Joe Moyal in retirement, still thinking about quantum theory



Private collection.

What is the measure of a scientist's life? Some would say the accolades, the recognition of scientific peers, and the adoption and use made of his original work. Joe Moyal published only 36 papers, a small total in relative terms, but most were fundamental works.

It is possible to follow their reception through the cited references of the `Web of Science'. `The general theory of stochastic population processes', of 1962, follows a high rising curve, as does `Theory of ionization fluctuations', of 1955, and his last major research paper, `Particle populations and number operators in quantum theory', of 1972. But none pass unnoticed or unrecognized.

It is, however, Joe's earliest paper, 'Quantum mechanics as a statistical theory', that has reverberated with increasing force and relevance to the present day. Its pattern of progress in the citation data of 'Web of Science', provides an index both of the evident expansion of the physics community and its significant diversification across these past 50 years. At the same time, it offers significant testimony to the paper's remarkable contribution to a raft of ranging and important developments in physics which Joe Moyal himself could never have anticipated.

Initially, in the small scientific community of 1949 and into the early 1950s, the paper's adoption was slow: three citations in 1949, four to five in the early 1950s, and on through fluctuating numbers to 10 in 1965 and 19 in 1969. The period through the 1970s to the 1980s saw expanding use, 24 cited references in 1977, 28 in 1982, and rising through further numbers into the 30s during the 1990s reaching 69 in 2001. By 2003, however, 'Quantum mechanics as a statistical theory' had built up a total of 980 cited references. By 2005, a further citation explosion had taken place and by mid-year the cumulative citation count for the paper had soared to 1,220. In April 2006, it reached 1,245. At this point nearly 35% of all citations published of a 1949 paper came from the years 1999–2006 and the citations rise with every month.

In the now far-flung scientific community from Russia, Yugoslavia, Slovenia, Hungary, Poland to Costa Rica and Brazil; with Europe, Britain, India, Japan, North and South America and the Antipodes in between, the 'Moyal bracket', the 'Moyal equation', the 'Moyal star product', the 'Moyal formula', `Moyal quantum', 'Moyal planes', the 'Wigner-Moyal' (stated at times as the `Weyl-Wigner-Moyal') formalism, and 'Moyal algebra' have found high resonance. In their mathematical and physics applications and influence, they reach into a stream of research and publications in quantum mechanics, quantum field theory, phase space, solid state physics, string theory, cosmology, quantum chaos, probability distributions, optics, tomography, diagrammatic techniques, deformation quantization, atomic systems, spectral line shape calculations, teleportation technology, and even brain research.

Defining quantum physics as ‘a physics of information’ as Moyal Medallist in 2001, Professor Gerard Milburn of the Centre for Quantum Computer Technology at the University of Queensland, averred: `Moyal quantum is also opening doors for the development of research in computation and communication’.[1]

In the huge literature opened by the ‘Web of Science’, there are many varied expressions of the impact of Joe’s work. Delivering a paper on ‘Wigner, Moyal, and Precursors to Canonical Coherent States’, at the Wigner Centennial Conference in Hungary in 2002, Professor John Klauder of the University of Florida’s Department of Physics and Mathematics, recalled:

> In 1957 while studying for my PhD at Princeton, I had run across the paper of Moyal — and like many others before and since — really appreciated what a fine paper it was … It is certainly the case that some classic papers of the past contain far more than was recognized at the time they were written. It is safe to say that Moyal’s classic paper on the Wigner function and its application to a completely phase space description of quantum mechanics, is just such a paper! … Although it was not recognized at the time, one may say that Moyal implicitly established the essence of the resolution of unity appropriate to the family of canonical coherent states for an arbitrary normalized fiducial vector.[2]

In the same year, Robert Littlejohn of the Department of Physics and Mathematics, University of California, Berkeley, addressing the theme of ‘Quantum Normal Forms via Moyal Star Product and space distribution function’, declared: ‘The concept of the Moyal bracket and the usual product of classical mechanics have been precursors to the recent program of deformation quantization [while], even more contemporary, is an effort to review and extend the Moyal program as a tool to analyze situations involving noncommuting geometry.'[3]

From diverse backgrounds, José Gracia-Bondia from the Department of Theoretical Physics at Universidad de Complutense de Madrid, and Joseph Várilly at the Department of Mathematics Universidad de Costa Rica, from their collaborative work, attest to the singular importance of his paper and its stimulus to their work:

> Without dispute 'Quantum Mechanics as a statistical theory' is one of the great physical papers of the 20th century … There have been two main influences in which Moyal's work has become intensely relevant to today's mathematical and theoretical physics and we have been fortunate to participate in both.
>
> The first one is in connection with the Moyal formalism for quantum mechanics. On our table rests a copy of the manuscript by Dirac in the *Review of Modern Physics, 1945,* in which the new viewpoint is mentioned probably for the first time. We discovered its charms in the mid-eighties, and worked both on its physical and its mathematical aspects between 1984 and 1991 … before turning to different matters. Some of our work in this period, like the one on "Moyal representation for spin", *Annals of Physics* (1989)[4] arguably broke new ground, and has received a fair number of citations, particularly from people working on quantum optics …
>
> Now, in 1999, a second coming took place. There was an extraordinary paper by Nathan Seiberg and Edward Witten,[5] the latter known as father of string theory, in which they argued that, in some natural and well-established limit, strings behave like objects living in a 'noncommutative space'. This turns out to possess the same mathematical structure as the space of observables in Quantum Mechanics on phase space, as formulated by Moyal. This opened a truly new fashion, in particular for the tentative 'quantum field theory on Moyal space'. Since then, the work by Moyal has received hundreds of further citations. Here

> we stress that, although the physical motivation and interpretation of the related theories is completely different, 'the underlying mathematics is the same'.[6]

From their interest in noncommutative spaces, Várilly and Gracia-Bondia, accordingly, have returned to their original interest, joining with Professors Bruno Iochum and Victor Gayral of the Centre de Physique Théorique, and Université de Provence, Marseilles, to write their collective paper, `Moyal Planes and Spectral Triples', in which they coin the term 'Moyalology'.[7]

'We are still working in related subjects/areas,' Várilly and Gracia-Bondia advise, `the mathematical beauty, richness and symmetry of the so-called Moyal product or Moyal algebra ensured it a place in the foreseeable future.'

Bruno Iochum's early initiative in this fertile collaborative research brought Joe Moyal around full circle to his early contact at the University in Marseilles with Professor Daniel Kastler. As Iochum wrote to the author in December 2005:

> As a spiritual son of Daniel Kastler, I began work in research on the interplay between physics and algebra and naturally later … on geometry in its noncommutative sense … It is by a strange path that I encountered the tracks of Joe Moyal. There is an old tradition in Marseilles to work on quantization by deformation, for instance, there Jean Marie Souriau was a master of this approach. I realized quite recently that this Moyal original idea could totally fit the noncommutive geometry setting (which has of course a much larger purpose) and I decided to get in touch with Joe Várilly and Jose Gracia-Bondia. In a way this pushes the Seiberg-Witten main approach in the broad landscape of quantum field theory on noncommutive spaces; a fact that probably Joe Moyal had not imagined!

Serendipity, circularity, chance, and growth all play their strangely meandering, yet purposive way in science. For 'science goes like a child', Iochum reflects. 'After its birth it grows and lives independently of its progenitors.' Many other papers fertilized by Joe's work proclaimed a kindred parental guidance.[8] Yet Várilly and Gracia-Bondia, go further:

> In retrospect, Joe Moyal corrected an excessive bent of the stick by the fathers of quantum theory. They were so much impressed by Heisenberg's uncertainty principle that they thought classical-looking mathematical descriptions were ruled out forever. In this regard they were wrong … The best, and only, example we can find of a parallel achievement in the kindred spirit in the whole story of Quantum Mechanics since the 1920s, is the discovery by Hohenberg and Kohn[9] that 'classical' electronic density is enough to determine the atomic structure, including exchange, which eventually gave rise to Density Functional Theory, and won Kohn his Nobel Prize. Their insight also runs against the 'intuition' bequeathed by Quantum Mechanics formalism as found in textbooks. Indeed, density functional theory is best and most naturally formulated in 'Moyal language'. But this, it appears, for the time being, has been rejected out of hand.[10]

Dr Cosmas Zachos, of Argonne's High Energy Physics Division, has set down a substantial overarching view of Joe Moyal's work:[11]

> Moyal's most celebrated paper remains the pioneering 1949 paper, well validated by posterity. In it, he established an independent formulation of quantum mechanics in phase space. This is the third, alternative, formulation of quantum mechanics, independent of the conventional Hilbert space, or path-integral formulations. It is logically complete and self-standing, and, by dint of its expression in phase space, like classical mechanics, it offers unique insights into the classical limit of quantum theory, and conversely, in quantization — the transition from classical to quantum mechanics. Because it enables useful retention of standard variables while endowing them with novel properties (such as noncommutativity, 'Moyal Brackets' and 'Moyal algebra'), 'Moyal' is nowadays freely used in physics as a loose adjective indicating noncommutativity, in ways that evoke 'Moyal

quantization', viz. 'Moyal plane', 'Moyal deformation', 'Moyal string field theory', 'Moyal approach', etc.

While the phase-space formulation grew out of important work by Hermann Weyl, Eugene Wigner, and especially Hilbrand Groenewold, the decisive formulation was pulled together by Moyal, in a grand synthesis of the scattered mathematical machinery into a confident interpretation of quantum mechanics as a statistical theory, with a systematic vision of its logical autonomy. Thus, the implicit injunction to go forth and apply this formulation to freely obtain results harder to reach in the conventional quantum mechanics picture is largely Moyal's.

Moyal systematically studied all expectation values of Weyl-ordered operators, and identified the Fourier transform of their moment-generating function (their characteristic function) to the Wigner Function. He then interpreted the subtlety of the 'negative probability' formalism based on this function, and reconciled it with the uncertainty principle and the diffusion of the probability fluid. Not least, he then recast the time evolution of the Wigner function through the deformation of the Poisson Bracket into the celebrated Moyal Bracket, a powerful construct of great impact in mathematical physics.

This formulation of quantum mechanics pioneered by Moyal serves in describing quantum transport processes in phase space. Such processes are of importance in quantum optics, nuclear and particle physics, condensed matter, the study of semi-classical limits of mesoscopic systems and phase transitions of classical statistical mechanics. It is the natural language to the study of quantum chaos and decoherence (of utility in, e.g. quantum computing), and provides crucial intuition in quantum mechanical interference problems, probability flows as negative probability backflows and measurements of atomic systems. The mathematical structure of the formulation is of relevance to Lie Algebras, martingales in turbulence, and string field theory. It has recently been retrofitted into M-theory advances linked to the noncommutative geometry hypothesized to underlie gravity at extremely short distances, and matrix models. In addition, it is significant outside physics, as for example in foundational work on wavelet methods in signal processing.

For Tony Bracken, Professor of Physics at the University of Queensland, `Moyal's work and the central role of the "Moyal bracket" has provided the basis for the mathematically profound notion of quantization as deformation, now an established area for important mathematical research.'[12]

Joe Moyal himself took a characteristically modest view of his own accomplishment. He knew, by the mid-1990s, of the significant flow-on of his work in quantum mechanics but, with his death in 1998, he was unable to realize the paper's rich cascade into burgeoning new disciplinary fields. Yet his postulation and the mathematics of `Quantum mechanics as a statistical theory' held the seeds of unanticipated applications and the fertilization of remarkable new approaches in quantum theory. He himself was well aware that he had not derived his original concept from Wigner's 1932 paper, of which he learnt only after his own formulation was made and, in this respect, he did not, in truth, `expand the ideas of Wigner', as current attributions traditionally declare, but presented his own formulation — with courteous reference to Wigner — when his paper came to print.

In a candid interview in 1979, however, he reflected, 'I felt that I was always on the edge. I always seemed to work on the fringes'.[13] Yet, with his alternative formulation of quantum mechanics, its reconciliation with the uncertainty principle, and his eloquent generalizations, he was in the direct line of descent from that cluster of highly creative scientists who, earlier in the century and notably the 1920s and early 1930s, had initiated a singular period of conceptual advance. The notion of the atom as object had been discarded and new mental constructs of mathematical expression had taken place. 'The idea', as one observer put it, 'had an austerity that went home to a certain type of mind … And it worked like none other in the history of science.'[14]

Joe's philosophy about the conduct of scientific research was, nonetheless, modest and straightforward:

> There are two types of reward, quite apart from the material rewards from doing work in science and technology, and one of the rewards is that it's just fun to be doing something new, to discover new things: what could be more stimulating and amusing! It's a lot of hard work but it's rewarding in itself. If one thinks about what sort of material rewards one will get for one's work and will one be appreciated by all and sundry, what prizes are you going to get from your work, who is going to read it, will you be upgraded in your profession, will you be elected to this or that body? If you think of this then you get nowhere, you get discouraged. You get bored with the whole procedure. But the other reward is that it's something permanent if you do good work. If you keep in mind that what you are doing may or may not prove valuable and you are looking for new discoveries or advancing the frontiers of knowledge in your area, you can pass the time to distinguish what is good or bad in your work. You never know how good it is going to be. Even one's errors are useful. One's mistakes can be valuable to some later scholars who, in discovering them, can lead to something new.[15]

His attitude to 'elitist science' and to election to professional bodies was clearly coloured by this straightforward view. It derived, too, from his quintessentially maverick style. He lacked any great respect for hierarchies and disliked the `old boy clubiness' and 'cultivating' often associated with election to royal societies and academies. Resultingly, he would never respond to overtures that sought to enrol him in the scientific elite. This, the goal of many men, he eschewed. He remained an occupant of the 41st chair,[16] those who, while as distinguished as many within the Academy, nevertheless remain outside.

Yet, as Henri Poincaré once pronounced on scientific thought ,'its genesis is an activity in which the human mind seems to borrow least from the external world', while the great G.H. Hardy judged the gift of mathematics 'one of the most specialized talents, and of all the arts and sciences, the most austere and remote'. Like Richard Feynman, whom he much admired, Joe drew a distinction between science and art. 'The problem posed for the scientist, different from the artist,' Feynman explained succinctly, `is to imagine something that you have never seen that is consistent in every detail from what has already been seen, and that is different from what has been thought.'[17] For men like Feynman and Joe Moyal, both bilingual in physics and mathematics and furnishing physicists with new conceptual tools, it was also essential that their work have practical applications. Again, Joe expressed it in a simple vocabulary: `Science and scholarship are really different from art and literature in that they are a process, and the contributions that one makes to this — the best of one's work — flow into that process and remain part of the "weave" in the development and beauty of science and technology.'[18]

Intermittently, from 1978, Joe embraced the process, returning 'to review' the Wigner/Moyal formalism:

> I worked out a new approach to the whole problem, which resolved some of the difficulties. I wanted to look at the later work, all I could lay my hand on, and when I started looking at it I remembered all the unsolved questions which had occurred to me at the time I wrote my two papers. Then I found, much to my surprise, that of the people who had done the work, none had addressed themselves to the difficulties and the things where I would have wanted to extend the formalism to, and where I hadn't succeeded at the time. So I took a fresh look at it and found that, by examining it from a somewhat different point of view, you could develop the whole theory as I had developed it and also extend it to include the theory of spin, because the particular thing that wasn't included in the original theory, is spin which is thought of as a distinctly non-classical element in quantum theory. But of course this is false. You could introduce spin into classical or quasi-classical theory and the Wigner/Moyal formalism lends itself rather well to the description of quantum mechanical particles with spin. There is no difficulty. In this

> recent work I found a new way of getting at the basic elements of the theorem and if you do that you painlessly get the generalization of particles with spin.[19]

He had also attempted to carry his ideas further and get a generalization to relativistic quantum theory or special relativistic quantum theory, but admitted defeat. Only his new method of doing the work `by the theory of group representations', he added, was conceptually complete. Now it was `only a matter of writing it out in detail'. But this was never completed.[20]

Overall, the poet Rainer Maria Rilke, a favourite of Joe's, catches the spirit of his scientific work:

> I live my life in growing orbits,
> Which move out over the things of the world,
> Perhaps I can never achieve the last,
> But that will be my attempt.
> And I still don't know if I am a falcon,
> Or a storm, or a great song.[21]

After his retirement from Macquarie University in 1978, Joe Moyal had two more decades of richly reflective and contented life. E.J. Bell, in his famous book *Men and Mathematics,* observes that highly creative mathematicians have long been displayed as `slovenly, absent-minded dreamers totally devoid of common sense'. Rather, he contends, that 'as a group the great mathematicians have been men of all-round ability, vigorous, alert, keenly interested in many things outside mathematics.' For his part, Joe remained a high activist well beyond his chronological years, scuba diving and snorkelling around the Great Barrier Reef into his late 70s, travelling and camping with Ann in a Toyota truck in Queensland's beautiful Daintree Forest, reading eclectically in science and history — *Nature* and *History Today* always close at hand — a cinema buff and wine connoisseur, a man keenly engaged in international politics and literature who retained an extraordinary knowledge and interest until the end of his days. Living for a time in retirement in Sydney, he returned in 1981 to reside permanently in Canberra.

Figure 8.2. Joe camping in Queensland, 1984

Private collection

Like many academic couples who had different schedules and, at times, geographies, the Moyals had divided interludes and grew accustomed to occupying different houses.[22] Yet writing in her autobiography of their marriage, Ann set down: `Across my life of scholarship and action, I know one thing: the thread that has made it buoyant and persistent is a complex and lasting love … We enjoyed a great affinity and, in our different ways, nourished and stimulated each other. Loving, good talk, a rich intellectual life, protection and support, journeyings and lovely places, encouragement for me in all I did, yielded a fulfilling diet.' [23]

There had been a marked series of flights in Joe Moyal's life — from his own country, then Palestine; from France; from Northern Ireland; from Britain to Australia; from Australia to the United States, and a final flight from America back to Australia. In his manner and kind, he belonged to several soils. But he found his anchor at last in Canberra. In his last years, his life came full circle. His son and daughter visited him from America, and a distinguished cousin, Shmuel Moyal, offspring of the long line of Moyal judges, lawyers, and diplomats in old Palestine and Israel, was appointed to Canberra in 1995 as Israel's Ambassador to Australia.

Ben-Gurion had been right about Jewish talent. Jews of the diaspora had excelled in mathematics and the growth of physics. José Enriques Moyal, it could be claimed, had a unique distinction. He was, arguably, the first `Israeli' mathematician emerging from that country's Turkish/Palestinian background to attain international recognition; and, with an honorary Doctorate of Science conferred on him in 1997 at The Australian National University on the grounds 'of his distinguished creative achievement as a scholar in mathematical statistics and mathematical physics', he had the further distinction of being the first `Israeli' to gain high scientific prominence in Australia.

Joe Moyal died in Canberra on 22 May, 1998, a few months before his 88th birthday. Yet, as one saying declares, `he who leaves has never truly left'. In his ends were his beginnings. His funeral service was presided over with the traditional Hebrew words of passage by his cousin, Ambassador Moyal.

With hindsight and in overview, then, it can be said that the lives of unique individuals, drawn from every scientific arena who have made original and enduring contributions in their work and who, through their enterprise, research and interactions have influenced and intersected with wide disciplinary communities, can make an edifying and important contribution to the ever evolving story, and the history, of science.

Figure 8.3. Portrait of Ann and Joe Moyal, Canberra, 1995

Photograph by Heide Smith

ENDNOTES

[1] G. J. Milburn, 'Communication and Computation in a Quantum World'. Second Annual Moyal Lecture, Macquarie University, June 2001.

[2] Professor John Klauder, Proceedings of Wigner Centennial Conference, Paper no 55. Klauder also refers to Moyal's 'classic paper' in his (with E.C.G. Sudarshan) *Fundamentals of Quantum Optics* , W.A. Benjamin Inc., 1968.

[3] Robert Littlejohn, 'Quantum Normal Forms by Moyal Star Product', Google source. `A sizable impact in this context was made by two remarkable papers by F. Bayen et al., *Annals of Physics*, (1978), vol. 111, pp. 61 and 111... They took Moyal's ideas and refashioned them into a new mathematical paradigm'. Information from Várilly and Gracia-Bondia, in communication to the author, 31 December 2005.

[4] *Annals of Physics (1989),* vol. 109, pp. 107–48.

[5] `String theory and noncommutative geometry,' *High Energy Physics* (Electronic Journal), 9909 (1999) 032.

[6] Communication to the author, ibid.

[7] Victor Gayral, José M. Gracia-Bondia, Bruno Iochum, Thomas Schucker and Joseph Várilly, `Moyal Planes and Spectral Triples', delivered at the Abdus Salam International Centre for Theoretical Physics, Miramare-Trieste, September 2003, *Commun. Math. Phys*. 2004, vol. 246, pp. 569–623.

[8] As one example, C. Tabisz, B.R. McQuarrie, and T.A. Osborn (Department of Physics, University of Manitoba), in their paper, 'Moyal Semiclassical Quantum Dynamics for Atomic Systems', *Physical Review A*, 1998, vol. 58 (4), pp. 2944–61, claim that from their several years research working on the Lennard-Jones potentials with model helium, neon and argon, their results 'provide a first demonstration of the practicality and usefulness of Moyal quantum

mechanics in the analysis of realistic atomic systems'. Importantly, Jans Peder Dahl, Professor of Chemical Physics at Lyngby (Copenhagen), is one of the pioneers of the use of the Moyal approach in atomic physics.

[9] *Physical Review B* (1964), vol. 136, p. 864.

[10] Communication to the author, 31 December, 2005.

[11] Communication to the author 3 October, 2005. In addition to his part in *Quantum Mechanics in Phase Space*, *op. cit.*, Cosmas Zachos is author and co-author of a number of papers relating to 'Moyalology', notably: 'Deformation Quantization. Quantum Mechanics Lives and Works in Phase Space', *International Journal of Modern Physics* A, 2002, vol. 17(3), pp. 297–316; (with T. Curtright and D. Fairlie), 'Features of Time-Independent Wigner Functions', *Phys. Rev.* D58, 1998, 025002; (with T. Curtright), 'Wigner Trajectory Characteristics in Phase Space and Field Theory', *J. Phys. A32*, 1999, pp. 771–9; (with T. Curtright and T. Uematsu) 'Generating All Wigner Functions', *Jour. Math. Phys.*, 2001, vol. 42, pp. 2396–415.; (with T. Curtright), 'Negative Probability and Uncertainty Relations', *Mod. Phys. Letters*, 2001, pp. 2381–5, and (with T. Curtright), 'Deformation Quantization of Superintegrable Systems and Nambu Mechanics', *New Jour. Phys.*, 2002, vol. 4, 83, 182.16.

[12] Communication from Professor Bracken to the author January 2006.

[13] Interview with Ann Moyal, 1979 *op. cit*.

[14] John de la Mothe, *C.P. Snow and the Struggle for Modernity*, University of Texas Press, Austin, 1992, p. 121.

[15] Interview 1979, *op. cit*.

[16] The concept of the 41st chair arises from the French Academy of Belles Lettres who, from their beginnings, decided that only a cohort of 40 could qualify as members and emerge as 'immortals'.

[17] R Feynman, *The Meaning of It All*, Helix Books, Addison-Wesley, Reading Massachusetts, p. 23.

[18] Interview with Ann Moyal, 1988, *op. cit*.

[19] Interview, 1979, *op. cit*.

[20] Joe Moyal bequeathed his manuscript material to his former student and colleague, Professor Peter Brockwell, Department of Statistics, University of Colorado, USA.

[21] 'I Live My Life' from *The Rag and Bone Shop of the Hearth. Poems for Men*, Robert Bly, James Hillman and Michael Meade (eds), Harper, Perenial, 1993, p. 421.

[22] Ann was Director of the Science Policy Research Centre, Griffith University, Queensland, from 1977–80.

[23] *Breakfast with Beaverbrook, op. cit*. p. 213.